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The fractional discrete model of COVID-19: solvability and simulation

Research Article

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Abstract: In this paper, we will discuss a discrete fractional order of covid-19 model and give results for existence and condition to ensure the disappearance of the disease.

MSC: 34N05; 39B82

Keywords: fractional order difference systems • Existence and uniqueness • Covid-19 models.

1. Introduction

After the spread of the Corona pandemic, many mathematicians made models to try to analyze the spread of this epidemic see ([1],[2],[3],...).

Recently modeling several chemical and physical phenomena have broadly been carried out using the theory of Fractional-order Difference Systems (FoDSs). The definitions of the fractional order deference operators arranged in order in [9] and later the development of the of som properites was done, while stability was studied in the commensurate order case in [4] and [5] and the incommensurate case in [6] and [7].

In this work, we will study the discret fractional model of one of the models of Covid 19 and the study of existence and uniqueness, and then we will present condition to ensure the disappearance of the epidemic. We will focus

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here on the following model:

$$\begin{cases} \frac{dS}{dt} = & \Lambda - (\delta + \lambda) S(t), \\ \frac{dE}{dt} = & \lambda S(t) - ((1-\varphi)\omega + \varphi\rho + \delta + \varepsilon_1) E(t), \\ \frac{dI}{dt} = & (1-\varphi)\omega E(t) - (\sigma_1 + \delta + \varsigma_1 + \tau) I(t), \\ \frac{dA}{dt} = & \varphi\rho E(t) - (\sigma_2 + \delta) A(t), \\ \frac{dQ}{dt} = & \varepsilon_1 E(t) - (\delta + \eta_1 + \varepsilon_2) Q(t), \\ \frac{dH}{dt} = & \tau I(t) + \varepsilon_2 Q(t) - (\delta + \eta_2 + \varsigma_2) H(t), \\ \frac{dR}{dt} = & \sigma_1 I(t) + \sigma_2 A(t) + \eta_1 Q(t) + \eta_2 H(t) - \delta R(t), \\ \frac{dM}{dt} = & m_1 I(t) + m_2 A(t) - m_3 M(t), \end{cases} \tag{1}$$

where $\lambda = \frac{\zeta_1 (I + \psi A)}{N} + \zeta_2 M$, ($\zeta_1, \psi, \zeta_2$ is selected to check $\lambda < 1$ during the time period we are interested in). The previous system is subject to the following nonnegative initial conditions:

$$\begin{cases} S(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \\ A(0) = A_0, \quad Q(0) = Q_0, \quad H(0) = H_0, \\ R(0) = R_0, \quad M(0) = M_0. \end{cases}$$

.The biological description of the parameters involved in COVID-19 model (1) is given in (2 and 3).

Variable	Description
$S(t)$	Susceptible class
$E(t)$	Exposed class
$I(t)$	Infected with clinical symptoms
$A(t)$	Asymptomatically infected
$Q(t)$	Quarantined class
$H(t)$	Hospitalized
$R(t)$	Recovered
$M(t)$	The environmental viral load due to the infected people

(2)

and

Parameter	Description
Λ	Recruitment rate
δ	Natural death rate
ω	Incubation period
ρ	Incubation period
σ_1	Infected recovery rate
σ_2	Asymptomatic recovery rate
η_1	Quarantined recovery rate
η_2	Recovery rate of the hospitalized
τ	Rate of moving from I to H class
ε_1	Quarantine rate of exposed individuals
ε_2	Hospitalization rate of Q individuals
ς_1	Infected disease death rate
ς_2	Hospitalized disease death rate
ζ_1	Contact rate
ζ_2	Disease transmission coefficient
m_1	Viral contribution to M by I class
m_2	Viral contribution to M by A class
m_3	Removalrate of virus from M
ψ	Transmissibility multiple

(3)

This model was studied in the fractional order continuous state in [1]. What interests us here is the following discrete fractional order case of this system.

In any case, this paper is divided as follows, in section 2 we will mention some initial concepts about fractional order difference calculus which we will need later in our study and then we will define the system that we will study, section 3 we study existence and uniqueness of the solution and then set a condition that guarantees the disappearance of the epidemic, section 4 includes numerical simulations that clarify what been studied.

2. Preliminaries

This section briefly introduces some basic definitions and preliminaries associated with discrete fractional calculus. Within the definitions below, the function f is defined on the set of the form $\mathbb{N}_a = \{a, a+1, a+2, ...\}$, where $a \in \mathbb{R}$.

Definition 2.1.
[5] Let $\alpha > 0$. Then, the $\alpha^{th}-$fractional sum, $\Delta_a^{-\alpha}$, of a function $f : \mathbb{N}_a \to \mathbb{R}$ is defined by:

$$\Delta_a^{-\alpha} f(t) := \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} f(s), \quad \text{for } t \in \mathbb{N}_{a+\alpha}, \tag{4}$$

where $\Gamma(.)$ is the Euler's gamma function.

Definition 2.2.
[5] Let $\alpha > 0$, $\alpha \notin \mathbb{N}$. Then, the $\alpha^{th}-$order Caputo fractional difference of a function f is defined by:

$${}^C\Delta_a^{\alpha} f(t) := \Delta_a^{-(n-\alpha)} \Delta^n f(t) = \frac{1}{\Gamma(n-\alpha)} \sum_{s=a}^{t-(n-\alpha)} (t-s-1)^{(n-\alpha-1)} \Delta^n f(s), \quad t \in \mathbb{N}_{a+n-\alpha}, \tag{5}$$

where $n = [\alpha] + 1$.

Proposition 2.1.
[5] Let $0 < \alpha \leq 1$, and f is defined on $\mathbb{N}_a$. Then

$$\Delta_{a+(1-\alpha)}^{-\alpha} \, {}^C\Delta_a^{\alpha} \, f(t) = f(t) - f(a), \quad \forall t \in \mathbb{N}_a. \tag{6}$$

Based on the two previous definitions, the $\alpha^{th}-$order Caputo fractional difference system associate to the system (1) is written as follows:

$$\begin{cases} {}^C\Delta_0^{\alpha} S(t) = & \Lambda - (\delta + \lambda) S(t-1+\alpha), \\ {}^C\Delta_0^{\alpha} E(t) = & \lambda S(t-1+\alpha) - ((1-\varphi)\omega + \varphi\rho + \delta + \varepsilon_1) E(t-1+\alpha), \\ {}^C\Delta_0^{\alpha} I(t) = & (1-\varphi)\omega E(t-1+\alpha) - (\sigma_1 + \delta + \varsigma_1 + \tau) I(t-1+\alpha), \\ {}^C\Delta_0^{\alpha} A(t) = & \varphi\rho E(t-1+\alpha) - (\sigma_2 + \delta) A(t-1+\alpha), \\ {}^C\Delta_0^{\alpha} Q(t) = & \varepsilon_1 E(t-1+\alpha) - (\delta + \eta_1 + \varepsilon_2) Q(t-1+\alpha), \\ {}^C\Delta_0^{\alpha} H(t) = & \tau I(t-1+\alpha) + \varepsilon_2 Q(t-1+\alpha) - (\delta + \eta_2 + \varsigma_2) H(t-1+\alpha), \\ {}^C\Delta_0^{\alpha} R(t) = & \sigma_1 I(t-1+\alpha) + \sigma_2 A(t-1+\alpha) + \eta_1 Q(t-1+\alpha) + \eta_2 H(t-1+\alpha) - \delta R(t-1+\alpha), \\ {}^C\Delta_0^{\alpha} M(t) = & m_1 I(t-1+\alpha) + m_2 A(t-1+\alpha) - m_3 M(t-1+\alpha), \end{cases} \tag{7}$$

where $0 < \alpha < 1$ and $t \in \mathbb{N}_{1-\alpha}$.

Theorem 2.1.
[9] If there exists a positive definite and decrescent scalar function $V(t,x)$ such that

$${}^C\Delta_{t_0}^{\alpha} V(t, x(t)) \leq 0, \tag{8}$$

for all $t_0 \in \mathbb{N}_a$, then the trivial solution of (6) is uniformly stable.

3. Existence and uniqueness

Now, to show the existence and uniqueness we use fixed point theory and Picard Lindelöf method. To proceed, we may rewrite the system described in (7) in the following classical form:

$$\begin{cases} {}^{C}\Delta_{0}^{\alpha} X(t) = F((t-1+\alpha), X(t-1+\alpha)), \\ X(0) = X_0, \end{cases} \tag{9}$$

where $t \in \mathbb{N}_{1-\alpha}^{T_{\max}}$,($(T_{\max}-1+\alpha) \in \mathbb{N}$) the vector $X(t) = (S(t), E(t), I(t), A(t), Q(t), H(t), R(t), M(t))^T$ and the function $F(t, X(t))$ is defined as follows:

$$\begin{aligned} F_1(t,S) = &\ \Lambda - (\delta+\lambda) S(t), \\ F_2(t,E) = &\ \lambda S(t) - ((1-\varphi)\omega + \varphi\rho + \delta + \varepsilon_1) E(t), \\ F_3(t,I) = &\ (1-\varphi)\omega E(t) - (\sigma_1 + \delta + \varsigma_1 + \tau) I(t), \\ F_4(t,A) = &\ \varphi\rho E(t) - (\sigma_2 + \delta) A(t), \\ F_5(t,Q) = &\ \varepsilon_1 E(t) - (\delta + \eta_1 + \varepsilon_2) Q(t), \\ F_6(t,H) = &\ \tau I(t) + \varepsilon_2 Q(t) - (\delta + \eta_2 + \varsigma_2) H(t), \\ F_7(t,R) = &\ \sigma_1 I(t) + \sigma_2 A(t) + \eta_1 Q(t) + \eta_2 H(t) - \delta R(t), \\ F_8(t,M) = &\ m_1 I(t) + m_2 A(t) - m_3 M(t), \end{aligned} \tag{10}$$

To do so we proceed in the following manner. Using initial conditions $(X(0))$ and Proposition 3, we transform the system (7) into the following sum equations:

$$\begin{cases} S(t) - S(0) = \Delta_{1-\alpha}^{-\alpha}(\Lambda - (\delta+\lambda) S(t-1+\alpha)), \\ E(t) - E(0) = \Delta_{1-\alpha}^{-\alpha}(\lambda S(t-1+\alpha) - ((1-\varphi)\omega + \varphi\rho + \delta + \varepsilon_1) E(t-1+\alpha)), \\ I(t) - I(0) = \Delta_{1-\alpha}^{-\alpha}((1-\varphi)\omega E(t-1+\alpha) - (\sigma_1 + \delta + \varsigma_1 + \tau) I(t-1+\alpha)), \\ A(t) - A(0) = \Delta_{1-\alpha}^{-\alpha}(\varphi\rho E(t-1+\alpha) - (\sigma_2 + \delta) A(t-1+\alpha)), \\ Q(t) - Q(0) = \Delta_{1-\alpha}^{-\alpha}(\varepsilon_1 E(t-1+\alpha) - (\delta + \eta_1 + \varepsilon_2) Q(t-1+\alpha)), \\ H(t) - H(0) = \Delta_{1-\alpha}^{-\alpha}(\tau I(t-1+\alpha) + \varepsilon_2 Q(t-1+\alpha) - (\delta + \eta_2 + \varsigma_2) H(t-1+\alpha)), \\ R(t) - R(0) = \begin{array}{l} \Delta_{1-\alpha}^{-\alpha}(\sigma_1 I(t-1+\alpha) + \sigma_2 A(t-1+\alpha) + \eta_1 Q(t-1+\alpha) + \eta_2 H(t-1+\alpha) \\ -\delta R(t-1+\alpha)), \end{array} \\ M(t) - M(0) = \Delta_{1-\alpha}^{-\alpha}(m_1 I(t-1+\alpha) + m_2 A(t-1+\alpha) - m_3 M(t-1+\alpha)), \end{cases} \tag{11}$$

for $t \in \mathbb{N}_{a+1-\alpha}^{T_{\max}}$. Using (10) and the definition of $\Delta_a^{-\alpha}$ in (11), we obtained the state variable in terms of $F_i(t, X(t))$, where $i = 1 \cdots 6$.

$$\begin{cases} S(t) = & S(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_1(s-1+\alpha, S(s-1+\alpha)), \\ E(t) = & E(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_2(s-1+\alpha, E(s-1+\alpha)), \\ I(t) = & I(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_3(s-1+\alpha, I(s-1+\alpha)), \\ A(t) = & A(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_4(s-1+\alpha, A(s-1+\alpha)), \\ Q(t) = & Q(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_5(s-1+\alpha, Q(s-1+\alpha)), \\ H(t) = & H(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_6(s-1+\alpha, H(s-1+\alpha)), \\ R(t) = & R(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_7(s-1+\alpha, R(s-1+\alpha)). \\ M(t) = & M(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_8(s-1+\alpha, M(s-1+\alpha)). \end{cases} \quad t \in \mathbb{N}_{1-\alpha}^{T_{\max}}. \tag{12}$$

The Picard iterations are given by the following equations:

$$\begin{cases} S_n(t) = & S(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_1(s-1+\alpha, S_n(s-1+\alpha)), \\ E_n(t) = & E(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_2(s-1+\alpha, E_n(s-1+\alpha)), \\ I_n(t) = & I(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_3(s-1+\alpha, I_n(s-1+\alpha)), \\ A_n(t) = & A(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_4(s-1+\alpha, A_n(s-1+\alpha)), \\ Q_n(t) = & Q(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_5(s-1+\alpha, Q_n(s-1+\alpha)), \\ H_n(t) = & H(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_6(s-1+\alpha, H_n(s-1+\alpha)), \\ R_n(t) = & R(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_7(s-1+\alpha, R_n(s-1+\alpha)). \\ M_n(t) = & M(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_8(s-1+\alpha, M_n(s-1+\alpha)). \end{cases} \quad t \in \mathbb{N}_{1-\alpha}^{T_{\max}}. \tag{13}$$

Corresponding to the form (12), and with the initial condition we have the following sum equation:

$$X(t) = X(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F(s-1+\alpha, X(s-1+\alpha)). \quad t \in \mathbb{N}_{1-\alpha}. \tag{14}$$

Lemma 3.1.
The function $F(t, X(t))$ defined in (10) satisfies the Lipschitz condition given by

$$\|F(t, X(t) - F(t, X(t))\| \leq \beta \|(X(t) - X(t))\|, \tag{15}$$

where

$$\beta = \max \left\{ \begin{array}{c} \|(\delta+\lambda)\|, \|((1-\varphi)\omega + \varphi\rho + \delta + \varepsilon_1)\|, \|(\sigma_1 + \delta + \varsigma_1 + \tau)\| \\ \|(\sigma_2 + \delta)\|, \|(\delta + \eta_1 + \varepsilon_2)\|, \|(\delta + \eta_2 + \varsigma_2)\|, \|\delta\|, \|m_3\| \end{array} \right\}. \tag{16}$$

Proof. Summarizing that $S(t)$ and $S^*(t)$ are couple functions, we reach

$$\|F_1(t,S) - F_1(t,S^*)\| = \|(\delta+\lambda)\,(S(t) - S^*(t))\|\,. \tag{17}$$

Taking into account

$$\beta_1 = \|(\delta+\lambda)\|\,, \tag{18}$$

one reaches

$$\|F_1(t,S) - F_1(t,S)\| \leq \beta_1 \,\|(S - S^*)\|\,. \tag{19}$$

Continuing in the same way, one gets

$$\begin{aligned}
\|F_2(t,E) - F_2(t,E^*)\| \leq &\quad \beta_2 \,\|(E - E^*)\|\,,\\
\|F_3(t,I) - F_3(t,I^*)\| \leq &\quad \beta_3 \,\|(I - I^*)\|\,,\\
\|F_4(t,A) - F_4(t,A^*)\| \leq &\quad \beta_4 \,\|(A - A^*)\|\,,\\
\|F_5(t,Q) - F_5(t,Q^*)\| \leq &\quad \beta_4 \,\|(Q - Q^*)\|\,,\\
\|F_6(t,H) - F_6(t,H^*)\| \leq &\quad \beta_6 \,\|(H - H^*)\|\,,\\
\|F_7(t,R) - F_7(t,R^*)\| \leq &\quad \beta_7 \,\|(R - R^*)\|\,,\\
\|F_8(t,M) - F_8(t,M^*)\| \leq &\quad \beta_8 \,\|(M - M^*)\|\,,
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
\beta_2 = &\quad \|((1-\varphi)\,\omega + \varphi\rho + \delta + \varepsilon_1)\|\,,\\
\beta_3 = &\quad \|(\sigma_1 + \delta + \varsigma_1 + \tau)\|\,,\\
\beta_4 = &\quad \|(\sigma_2 + \delta)\|\,,\\
\beta_5 = &\quad \|(\delta + \eta_1 + \varepsilon_2)\|\,,\\
\beta_6 = &\quad \|(\delta + \eta_2 + \varsigma_2)\|\,,\\
\beta_7 = &\quad \|\delta\|\,,\\
\beta_8 = &\quad \|m_3\|\,.
\end{aligned} \tag{21}$$

From (19-20), we find that the kernels $F_i, 1 \leq i \leq 8$ is satisfying the Lipschitz condition, moreover if $\beta_i < 1$, for $1 \leq i \leq 8$ then the kernel F_i is contraction. □

Theorem 3.1.
Assuming we have (16), then there exist a unique solution to the system (7) if

$$\beta \left|(T_{\max} - a)^{(\alpha)} - (1-\alpha)^{(\alpha)}\right| < 1. \tag{22}$$

Proof. The solution to the system (9) is

$$X(t) = P(X(t)), \tag{23}$$

where, P is the Picard operator defined by

$$P(X(t)) = X(0) + \frac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} F(s-1+\alpha, X(s-1+\alpha)). \tag{24}$$

Further, we have

$$\begin{aligned} \|P(X_1(t) - P(X_2(t))\| &= \left\| \tfrac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} (F(s-1+\alpha, X_1(s-1+\alpha)) \right. \\ &\quad \left. - F(s-1+\alpha, X_2(s-1+\alpha)))\right\|, \\ &\leq \tfrac{1}{\Gamma(\alpha)} \sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} \|(F(s-1+\alpha, X_1(s-1+\alpha)) \\ &\quad - F(s-1+\alpha, X_2(s-1+\alpha)))\|, \\ &\leq \tfrac{1}{\Gamma(\alpha)} \left(\sum_{s=1-\alpha}^{t-\alpha} (t-s-1)^{(\alpha-1)} \right) \max_{s \in \mathbb{N}_0^{t-\alpha}} \|(F(s-1+\alpha, X_1(s-1+\alpha)) \\ &\quad - F(s-1+\alpha, X_2(s-1+\alpha)))\|, \\ &\leq \tfrac{(T_{\max})^{(\alpha)} - (1-\alpha)^{(\alpha)}}{\Gamma(\alpha)} \beta \|(X_1(t) - X_2(t))\|. \end{aligned} \tag{25}$$

Since, $\frac{(T_{\max})^{(\alpha)} - (1-\alpha)^{(\alpha)}}{\Gamma(\alpha)} \beta < 1$, $(t \leq T_{\max})$ then, the operator P is a contraction, hence the system (9) has a unique solution in ℓ^∞. ∎

To evaluate the equilibrium let:
${}^C\Delta_0^\alpha S(t+1-\alpha) = {}^C \Delta_0^\alpha E(t+1-\alpha) = {}^C \Delta_0^\alpha I(t+1-\alpha) = {}^C \Delta_0^\alpha A(t+1-\alpha) = {}^C \Delta_0^\alpha Q(t+1-\alpha) = {}^C \Delta_0^\alpha H(t+1-\alpha) = {}^C \Delta_0^\alpha R(t+1-\alpha) = {}^C \Delta_0^\alpha M(t+1-\alpha) = 0$. System (7) become:

$$\begin{cases} \Lambda - (\delta + \lambda) S(t) = 0, \\ \lambda S(t) - ((1-\varphi)\omega + \varphi\rho + \delta + \varepsilon_1) E(t) = 0, \\ (1-\varphi)\omega E(t) - (\sigma_1 + \delta + \varsigma_1 + \tau) I(t) = 0, \\ \varphi\rho E(t) - (\sigma_2 + \delta) A(t) = 0, \\ \varepsilon_1 E(t) - (\delta + \eta_1 + \varepsilon_2) Q(t) = 0, \\ \tau I(t) + \varepsilon_2 Q(t) - (\delta + \eta_2 + \varsigma_2) H(t) = 0, \\ \sigma_1 I(t) + \sigma_2 A(t) + \eta_1 Q(t) + \eta_2 H(t) - \delta R(t) = 0, \\ m_1 I(t) + m_2 A(t) - m_3 M(t) = 0, \end{cases} \tag{26}$$

For which we get the DFE as follows:

$$(S_0, 0, 0, 0, 0, 0, 0, 0) = (\frac{\Lambda}{\delta}, 0, 0, 0, 0, 0, 0, 0). \tag{27}$$

The basic reproduction number R_0 is given by ([1]):

$$R_0 = \frac{k_1 \delta m_3 \zeta_1 \varphi \rho \psi + k_2 \delta m_3 \zeta_1 \omega - k_2 \zeta_2 \varphi \omega \Lambda m_1}{k_0 k_1 k_2 m_3 \delta} + \frac{k_1 \zeta_2 \rho \varphi \Lambda m_2 + k_2 \zeta_2 \omega \Lambda m_1 - k_2 \delta \zeta_1 \varphi \omega m_3}{k_0 k_1 k_2 m_3 \delta}. \tag{28}$$

where

$$\begin{aligned} k_0 &= (1-\varphi)\omega + \varphi\rho + \delta + \varepsilon_1, \\ k_1 &= \sigma_1 + \delta + \varsigma_1 + \tau, \\ k_2 &= \sigma_2 + \delta, \\ k_3 &= \delta + \eta_1 + \varepsilon_2, \\ k_4 &= \delta + \eta_2 + \varsigma_2. \end{aligned} \tag{29}$$

Theorem 3.2.
If

$$R_0 < 1, \tag{30}$$

then the COVID-19 free equilibrium $(S_0, 0, 0, 0, 0)$ *is globally asymptotically stable.*

Proof. Lyapunov function is commonly used to prove the global stability of the Disease Free Equilibrium of [7]. Taking in consideration the formed Lyapunov function of the type:

$$L(t) = \theta_1 E + \theta_2 I + \theta_3 A + \theta_4 M. \tag{31}$$

According to [1], if

$$\theta_1 = m_3\delta, \;\; \theta_2 = \frac{m_3\delta\zeta_1 + \Lambda m_1\zeta_2}{k_1}, \;\; \theta_3 = m_3\psi\delta\zeta_1 + \Lambda m_2\zeta_2 k_2, \;\; \theta_1 = \Lambda\zeta_2. \tag{32}$$

then

$$^{C}\Delta_0^{\alpha} L(t) \;\; \leq k_0 m_3 \delta \left(R_0 - 1\right) E, \tag{33}$$

Then $L(t)$ negative defined if $R_0 < 1$. According to Theorem 3 the pandemic will disappear. □

4. Numerical simulation

In this section we will perform numerical simulations to verify the results mentioned in the previous section. We take a population $(N(0))$ of 7315, divided as follows:

$$\begin{cases} S(0) = 4000, \;\; E(0) = 3000, \;\; I(0) = 40, \\ A(0) = 10, \quad\;\; Q(0) = 18, \quad H(0) = 7, \\ R(0) = 100, \quad M(0) = 140. \end{cases}$$

And let:

$$\begin{aligned}
&\Lambda = 0.0038; \quad \eta_1 = 0.0006715; \quad \varsigma_2 = 0.0006715; \quad \psi = 0.0005856;\\
&\delta = 0.000612; \quad \eta_2 = 0.0006052; \quad \zeta_1 = 0.15; \quad \varphi = 0.4.\\
&\omega = 0.00129; \quad \tau = 0.00003061; \quad \zeta_2 = 0.00144;\\
&\rho = 0.00001305; \quad \varepsilon_1 = 0.00024; \quad m_1 = 0.000216;\\
&\sigma_1 = 0.0005253; \quad \varepsilon_2 = 0.00003; \quad m_2 = 0.000228;\\
&\sigma_2 = 0.000386; \quad \varsigma_1 = 0.000222; \quad m_3 = 0.0002276;
\end{aligned} \tag{34}$$

The following is a numerical simulation of the total number of infection in several alpha cases and by changing some parameters:

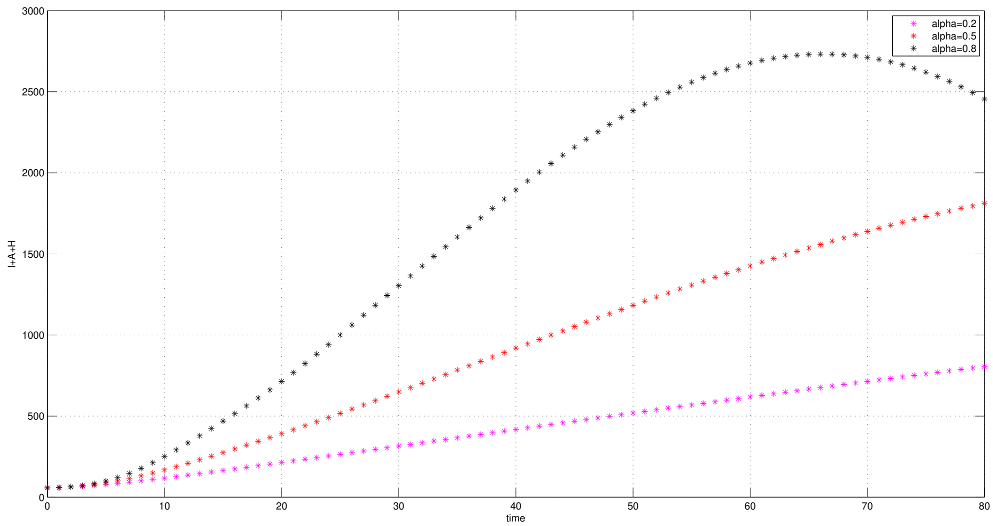

Figure 1. **A numerical simulation of the previous example.**

It can be seen that the fractional order does not change the behavior of the epidemic, but it gives us more flexibility to describe the spread of the disease.

5. CONCLUSIONS

In this papere, a discrete fractional order Covid-19 model is studied and some results of existence and global stability are given by using an appropriate Lyapunov function to ensure the disappearance of the disease in comfortable conditions.

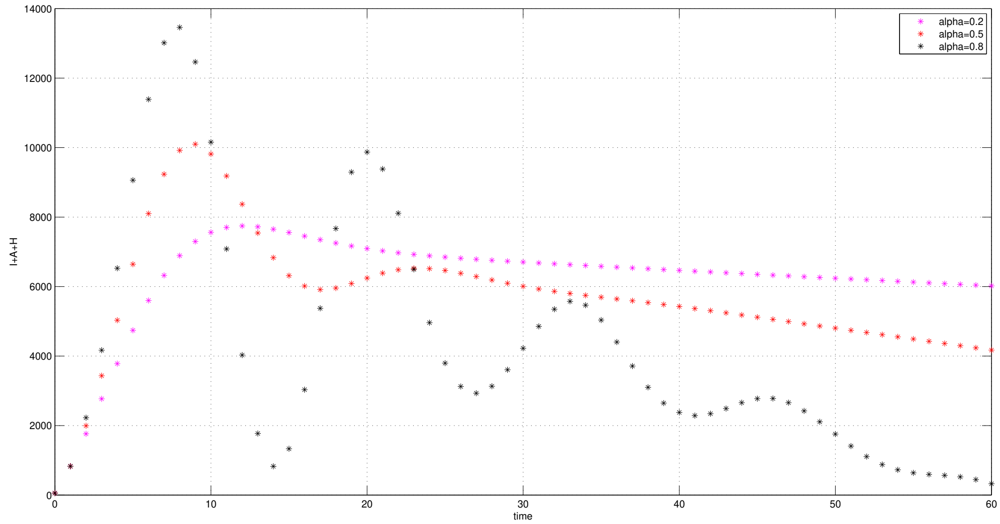

Figure 2. **A numerical simulation of the previous example by placing =0.129.**

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